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HEAT TRANSFER IN THIN LIQUID FILMS

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The objective of this summer research was to examine heat transfer in thin liquid films. A successful formulation was accomplished on the effect of electrostatic field on the stability of isothermal non-evaporating thin films and on the heat transfer in evaporating thin films. Here, the coupling of the electrostatic field with the fluid dynamics was accomplished through the interfacial boundary condition only. The weakness of this approach lies in the fact that the body forces due to the electrical field are ignored in the equation of motion and the work terms are not included in the conservation law for energy balance. Lastly, a general formulation including the electric field and Van der Waals forces between a solid surface and the fluid in the Navier Stokes equation was written but not applied to a particular geometry. A thorough literature search reveals that analysis that includes these effects has not been undertaken until now. The governing equations for the fluid as well as the electrical field are given for the previously mentioned two problems.

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FOREWORD

This report was prepared by the Aerospace Structures Information and Analysis Center (ASIAC), which is operated by CSA Engineering, Inc. under contract number F33615-94-C-3200 for the Flight Dynamics Directorate, Wright-Patterson Air Force Base, Ohio. The report presents the work performed under ASIAC Task No. T-29. The work was sponsored by the Structural Integrity Branch, Structures Division, Flight Dynamics Directorate, WPAFB, Ohio. The technical monitor for the task was Dr. Larry W. Byrd of the Structural Integrity Branch. The study was performed by Dr. Rama S. R. Gorla of Cleveland State University, under contract to CSA Engineering Inc.

This technical report covers work accomplished from May 1996 through August 1996.

1.0 INTRODUCTION

The long range objectives of this research are to identify and evaluate the heat transfer characteristics of evaporating thin liquid films.

The optimal design of a heat exchanger is the one that provides the highest heat transfer rate at the lowest overall cost. Derjaguin [1] introduced the concept of the "disjoining pressure" to describe the effect of Van der Waals dispersion forces in very thin films and their role in the dry-out of liquid films. Below a critical film thickness, a non-evaporating absorbed layer exists. As the liquid film increases in thickness, the disjoining pressure gradient enhances flow into the film which they suggested could increase the evaporation rate several fold. Potash and Wayner [2] experimentally determined two distinct regions of the evaporating extended meniscus on a vertical flat plate. Miller [3] studied the stability of liquid-vapor interfaces moving as a result of phase transformation or mass transfer. Das Gupta et al [4] used the Kelvin-Clapeyron change of phase heat transfer model to evaluate experimental data for an evaporating meniscus.

In the present study, attention is focused on the analysis of the following aspects of thin liquid films:

1. The effect of an electric field on the stability of a non-evaporating thin film.
2. The effect an of electric field on evaporating thin films.

The formulation of some of these problems will be summarized in this report.

2.0 EFFECT OF ELECTROSTATIC FIELD ON NON-EVAPORATING FILM RUPTURE

The behavior of a liquid film as it flows down an inclined plane was studied in references [5-10]. All these research workers ignored the London Van der Waals attractions between the solid surface and the liquid and therefore their results cannot be directly applied to the case of thin liquid films.

Our aim here is to address the question of how the thin liquid film and an electrostatic field interact. We are interested in the specific working regimes of the parameters, where it will be possible to stop a rupture (or dryout) of the film. This will be done by solving the equations of film motion in the presence of an electric field. Questions concerning the film stability will be addressed.

We consider the flow of a thin liquid film down an inclined plane under gravity. The plane is assumed to make an angle β with the horizontal. We choose x and y directions to be parallel and normal to the plane, respectively as shown in Figure 1. We assume that the characteristic thickness of the film to be d and the length scale parallel to the film to be L. The aspect ratio $\xi = d/L$. For a thin film, $\xi \ll 1$. The distance from the charged foil and the plane is H.

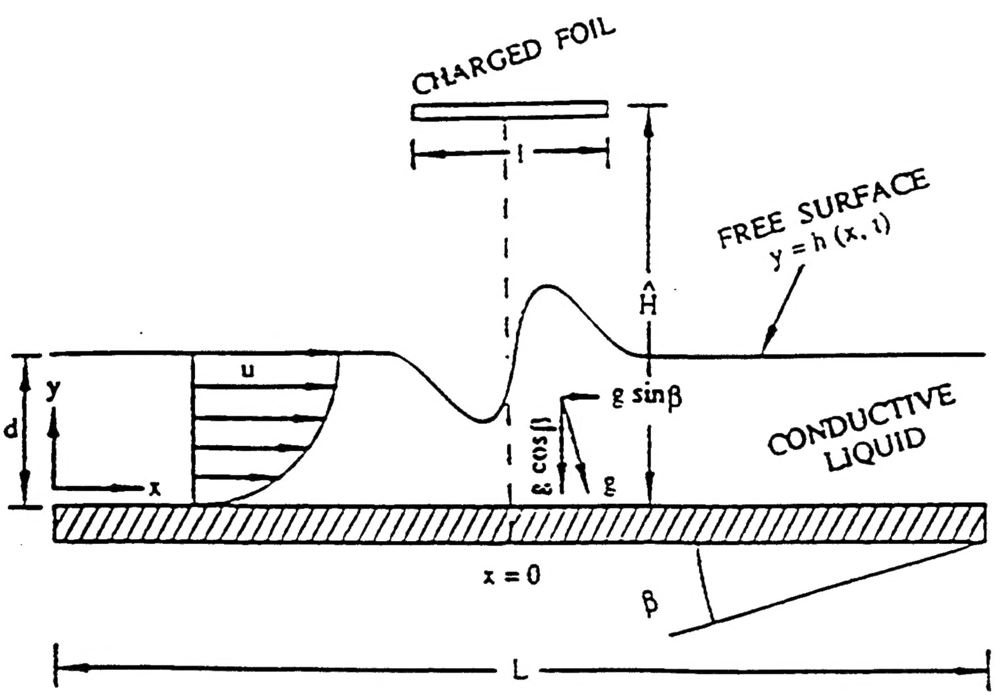

Figure 1. Flow Model for the Thin Film Flow

The electric field is determined by solving Laplace's equation.

$$\nabla^2\phi = 0 \tag{1}$$

where $\phi(x, y)$ is the electric potential. The boundary conditions are

$$\phi(x, H) = \Phi(x);\ \phi(x, 0) = 0 \tag{2}$$

Along $y = h(x, t)$ we have the boundary conditions that the tangential electric field and the normal displacement field are continuous. It may be noted that $y = h(x, t)$ is unknown, so that solution of the electrostatic problem is coupled to the dynamics of the film.

The liquid film is governed by the Navier-Stokes equations. The liquid layer is assumed thin enough that Van der Waals forces are effective and thick enough that a continuum theory of the liquid is applicable.

We assume that the liquid is incompressible. The governing equations and boundary conditions are made dimensionless by using the following scales: length in y-direction $\approx d$, in x-direction $\approx L$, velocity in x-direction $\approx U_0$, velocity in y-direction $\approx \xi U_0$, unit of time $\approx L/U_0$, unit of pressure ρU_0^2, and unit of electric field $\approx F$. We take ρ as the fluid density, ε_0 the dielectric constant and μ the viscosity. The continuity equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

The momentum equation becomes

$$\xi\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\xi\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\xi^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{Fr^2}\sin\beta - \xi\frac{\partial \psi}{\partial x} \tag{4}$$

$$\xi^2\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + \xi v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\xi}{Re}\left(\xi^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{Fr^2}\cos\beta - \xi\frac{\partial \psi}{\partial y} \tag{5}$$

In the above equations, u and v are the velocity components in x and y directions respectively, p is the pressure and ψ is the dimensionless potential function representing the van der Waal's forces. We follow Williams and Davis [10] and write a modified expression for ψ:

$$\psi = Ah^{-N} \tag{6}$$

where A is related to the Hamaker constant A' as $A = \dfrac{A'}{6\pi\rho U_0^2}$. We have introduced the Reynolds number, $Re = \rho U_0 d/\mu$ and the Froude number, $Fr = U_0/\sqrt{gd}$.

The boundary conditions along the solid plane wall are given by:

$$y = 0;\ u = v = 0 \tag{7}$$

At the fluid interface, we have the kinematic condition:

$$y = h(x,t) : \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = v \tag{8}$$

The continuity of tangential stress on the interface requires

$$y = h(x,t) : \left[1 - \xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]\left(\frac{\partial u}{\partial y} + \xi^2\frac{\partial v}{\partial x}\right) + 2\xi^2\frac{\partial h}{\partial x}\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right) = 0 \tag{9}$$

The continuity of normal stress at the interface $y = h(x,t)$ becomes

$$\frac{\xi^2}{Ca}\frac{\partial^2 h}{\partial x^2}\left[1 + \xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]^{-3/2} + Ah^{-N} = -\frac{Re}{2}p + K\left(\frac{1}{\varepsilon_f} - 1\right)[(E_n^v)^2$$

$$+ \varepsilon_f(E_t^v)^2] + \xi\left[\xi^2\left(\frac{\partial h}{\partial x}\right)^2 \frac{\partial u}{\partial x} - \frac{\partial h}{\partial x}\left(\frac{\partial u}{\partial y} + \xi^2\frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial y}\right] \times \left[1 + \xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]^{-1} \tag{10}$$

$K = \dfrac{\varepsilon_0 d F^2}{16\pi\mu U_0}$, where:

ε_f = dielectric constant of the fluid, dielectric constant for vapor assumed = 1

ε_0 = electrical permittivity of free space

$E^v_{n,t}$ = normal and tangential components of the electric field in the vapor at the interface

Ca = $2\mu U_0/\sigma$ is the capillary number

The left hand side of Eq. (10) includes the term Ah^{-N} to describe disjoining pressure effects explicitly in the interfacial boundary condition. There is still some question about whether this should be included. Williams and Davis [10] did not include this term in a similar analysis. This does not change the leading order values for u_1, v_1, p_1, etc. Equations (3) - (10) determine the motion of the liquid film. Our aim here is to solve for the stability of the liquid film while including the effect of Van der Waals forces and an applied electric field.

We now apply the long-wave theory to study the stability problem. When the layer is thinner than a critical value, small disturbances begin to grow. These waves have wavelengths much larger than the mean thickness of the layer. Defining a small parameter κ that is related to wave number of such disturbances, we may rescale the governing equations:

$$X = \kappa x;\ Y = y;\ \tau = \kappa t \tag{11}$$

we now assume the following expansions for the flow field:

$$u = u_0 + \kappa u_1 + \kappa^2 u_2 + 0(\kappa^3)$$

$$v = \kappa [v_0 + \kappa v_1 + \kappa^2 v_2 + 0(\kappa^3)]$$

$$p = \frac{1}{\kappa}[p_0 + \kappa p_1 + \kappa^2 p_2 + 0(\kappa^3)] \tag{12}$$

$$\psi_0 = \kappa\psi = 0(1) \text{ as } \kappa \to 0$$

Neglecting the variation of p with y and substituting expressions (12) into equations (1) - (10), we get

$$u_0 = \left[-\frac{Re}{Fr^2}\sin\beta + Re\cdot\xi\cdot\left(\frac{\partial p_0}{\partial X} + \frac{\partial \psi_0}{\partial X}\right)\right]\cdot\left[\frac{Y^2}{2} - hY\right] \tag{13}$$

$$v_0 = -Re\cdot\xi\cdot\left(\frac{\partial^2 p_0}{\partial X^2} + \frac{\partial^2 \psi_0}{\partial X^2}\right)\left[\frac{Y^3}{6} - \frac{hY^2}{2}\right] + \frac{Re}{2}\left[\frac{-\sin\beta}{Fr^2} + \xi\left(\frac{\partial p_0}{\partial X} + \frac{\partial \psi_0}{\partial X}\right)\right]\left(\frac{\partial h}{\partial X}\right)Y^2 \tag{14}$$

$$p_0 = \frac{2}{Re}\cdot\left(\bar{K}\left(\frac{1}{\varepsilon_f} - 1\right)[(E_n^v)^2 + \varepsilon_f (E_t^v)^2] - \frac{\xi^2}{\bar{C}a}\ \frac{\partial^2 h}{\partial X^2}\right) \tag{15}$$

where $\bar{K} = K\cdot\kappa;\ \bar{C}a = Ca/\kappa^3$.

Similarly, expressions u_1, v_1 and p_1 may be derived. Since these expressions are very long, they are not reproduced here.

Using equations (13) - (15) we may show that the leading order evolution equation for the film rupture is given by:

$$\begin{aligned}
&\frac{\partial h}{\partial \tau} - \frac{h^2}{2}\{-\frac{Re}{Fr^2}\sin\beta + Re\cdot\xi\left[\frac{2}{Re}\cdot\left(-\frac{\xi}{\bar{C}a}\cdot\frac{\partial^3 h}{\partial X^3} + \bar{K}\left(\frac{1}{\varepsilon_f} - 1\right)\frac{d}{dX}[(E_n^v)^2 + \varepsilon_f (E_t^v)^2]\right)\right.\\
&\qquad\left.-\frac{\kappa AN}{h^{N+1}}\cdot\frac{\partial h}{\partial X}\right]\}\frac{\partial h}{\partial X}\\
&= \frac{Re\xi h^3}{3}\cdot\{\frac{2}{Re}\cdot\left(-\frac{\xi}{\bar{C}a}\cdot\frac{\partial^4 h}{\partial X^4} + \bar{K}\left(\frac{1}{\varepsilon_f} - 1\right)\right)[(E_n^v)^2 + \varepsilon_f (E_t^v)^2]\\
&\qquad + \frac{\kappa AN}{h^{N+1}}\left[-\frac{\partial^2 h}{\partial X^2} + \frac{(N+1)}{h}\left(\frac{\partial h}{\partial X}\right)^2\right]\}\\
&+ \frac{h^2}{2}\cdot\frac{\partial h}{\partial X}\cdot\{-\frac{Re}{Fr^2}\sin\beta + Re\cdot\xi\left[\frac{2}{Re}\cdot\left(-\frac{\xi}{\bar{C}a}\frac{\partial^3 h}{\partial X^3} + \bar{K}\left(\frac{1}{\varepsilon_f} - 1\right)\frac{d}{dX}[(E_n^v)^2 + \varepsilon_f (E_t^v)^2]\right)\right]\\
&\qquad - \frac{\kappa AN}{h^{N+1}}\cdot\frac{\partial h}{\partial X}\}
\end{aligned} \tag{16}$$

subject to initial conditions:

$$h(X, 0) = F(X) \tag{17}$$

Equations (16) and (17) may be solved numerically in order to predict the stability characteristics.

3.0 EFFECT OF ELECTROSTATIC FIELD ON EVAPORATING FILMS

Here, we consider a thin liquid film joining an absorbed film and the meniscus of an evaporating interface under the influence of an electrostatic field. From the previous analysis, to a leading order of magnitude, we have

$$p_l - p_v = \frac{\bar{A}}{\delta^N} - \sigma\frac{d^2\delta}{dx^2}\left[1+\left(\frac{d\delta}{dx}\right)^2\right]^{-3/2} + \frac{\varepsilon_0}{8\pi}\left(\frac{1}{\varepsilon_f}-1\right)[(E_n^v)^2 + \varepsilon_f\cdot(E_t^v)^2] \tag{18}$$

In the above equation, p_l is the liquid pressure; p_v the vapor phase pressure; $\bar{A}$ the Hamaker constant (negative for a spreading liquid); δ the film thickness; ε_f the fluid dielectric constant and E the electric field.

Using the lubrication approximation for the liquid flow in the thin film we have

$$\mu_l\frac{\partial^2 u}{\partial y^2} = \frac{dp_l}{dx} \tag{19}$$

with boundary conditions given by

$$y = 0:\ u = 0 \text{ and } y = \delta:\ \frac{\partial u}{\partial y} = 0: \tag{20}$$

From equations (19) and (20) we may write

$$u = \frac{1}{\mu_l}\cdot\frac{dp_l}{dx}\cdot\left[\frac{y^2}{2}-\delta y\right] \tag{21}$$

The mass flow rate per unit width is given by

$$\Gamma = \rho_l\int_0^\delta u\,dy = -\frac{\delta^3}{3\upsilon_l}\frac{dp_l}{dx} \tag{22}$$

Following Wayner [47], the evaporative flux by means of theKelvin-Claperon equation may be written as

$$\dot{m} = a(T_{lv}-T_v) + b(p_l-p_v) = \frac{1}{\left(1+\frac{a\Delta h_m}{k}\delta\right)}\cdot[a(T_s-T_v)+b(p_l-p_v)]\ \cdot \tag{23}$$

where

$\dot{m}$ = evaporative flux

$a = 2\sqrt{\frac{M}{2\pi R T_{lv}}}\left(\frac{P_v M \Delta h_m}{R T_v T_{lv}}\right)$

$b = 2\sqrt{\frac{M}{2\pi R T_{lv}}}\left(\frac{V_l P_v}{R T_{lv}}\right)$

T_{lv} = temperature of liquid-vapor interface

T_v = temperature of the vapor

Δh_m = enthalpy of vaporization per unit mass

k = thermal conductivity of the liquid

T_{lv} is related to the surface temperature by the one-dimensional heat conduction equation $\frac{k}{\delta}(T_s - T_{lv}) = \dot{m}\Delta h_m$. Evaporative flux is related to the flow rate in the film through mass balance in the following form:

$$\frac{d\Gamma}{dx} = -\dot{m} \tag{24}$$

From equations (23) and (24), we may write

$$\frac{1}{3\upsilon_l}\frac{d}{dx}\left(\delta^3\frac{dp_l}{dx}\right) = \frac{1}{1+\frac{a\Delta h_m}{k}\delta}\left[a\Delta T + b\,(p_l - p_v)\right] \tag{25}$$

where $\Delta T = T_s - T_v$.

we define the following dimensionless variables:

$\delta_0^3 = -\frac{\bar{A}b}{a\Delta T}$ = reference thickness

$\pi_0 = \frac{a\Delta T}{b}$ = reference pressure

$\phi = \frac{p_l - p_v}{\pi_0}$ = dimensionless pressure difference

$$\iota = \sqrt{\frac{-\bar{A}}{\upsilon_\iota a \Delta T}} \quad = \quad \text{scale factor for X}$$

$$\xi = \frac{x}{\iota} \quad = \quad \text{dimensionless position}$$

$$\eta = \frac{\delta}{\delta_0}$$

$$\kappa = \frac{a \Delta h m \delta_0}{k} \quad = \quad \text{dimensionless thickness}$$

$$\varepsilon = \frac{\sigma \delta_0 b \upsilon_\iota}{(\bar{A})}$$

Dividing equation (18) by π_0 we get the dimensionless form:

$$\phi = -\frac{1}{\eta^N} - \varepsilon \frac{d\eta^2}{d\xi^2} + \Im(\xi) \tag{26}$$

where $\Im(\xi) = \left(\left(\frac{\varepsilon_0 F^2}{8\pi}\right) \cdot \frac{1}{\pi_0} \cdot \left(\frac{1}{\varepsilon_f} - 1\right)\right)[(\bar{E}_n^v)^2 + \varepsilon_f (\bar{E}_t^v)^2]$

where $\bar{E}_n^v = E_n^v / F$, $\bar{E}_t^v = E_t^v / F$

Equation (25) reduces to

$$\frac{1}{3} \frac{d}{d\xi}\left(\eta^3 \frac{d\phi}{d\xi}\right) = \frac{1}{(1 + \kappa\eta)} \cdot (1 + \phi) \tag{27}$$

Boundary conditions are:

$$\begin{gathered} \xi \to \infty : \eta \to 1 ; \phi \to -1 \\ \xi \to -\infty : \eta \to \infty ; \ \phi \to \phi_m \end{gathered} \tag{28}$$

Equations (26) - (28) may be solved to predict the effect of electrostatic field on the evaporating film heat transfer.

4.0 EFFECT OF ELECTRICAL FIELD ON EVAPORATING FILMS

In the previous two cases, it has been assumed that the coupling of the electrostatic problem to the fluid dynamics of the film is only through the boundary condition imposed at the liquid-vapor interface. Although this may be valid in the case of weak electrical fields, the case involving strong electrical fields (which is of practical interest) should consider the proper body forces in the momentum equation and the work terms in the energy equation.

References [11-20] consider the effect of electrical fields on single phase flows. A thorough literature search has indicated that the effect of electrical fields on evaporating films or two phase flows has not been investigated so far.

At the conclusion of the present task, there was not enough time to formulate this problem in detail. Therefore, only the governing equations for the problem are given here.

We assume that the fluid is incompressible. The current in the fluid is assumed to obey Ohm's law with the conductivity a function of temperature. The governing equations for the convective heat transfer in an electric field may be written as:

Mass:

$$\nabla \cdot \vec{V} = 0 \tag{29}$$

Momentum:

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla (\vec{V})\right] = -\nabla P + \mu \nabla^2 \vec{V} + q\vec{E} - \frac{E^2}{2}\nabla \varepsilon + \vec{F}_{Vdw} \tag{30}$$

where $\vec{F}_{Vdw}$ is the volumetric body force due to long range intermolecular (Van der Wall) forces.

Energy:

$$\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T = \alpha \nabla^2 T + \frac{\sigma E^2}{\rho C p} \tag{31}$$

where

ρ = density of the fluid

μ = viscosity

q = charge density

$\vec{E}$ = electric field

$\vec{V}$ = velocity field

$$P = p_l - \frac{E^2}{2}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T$$

p_l = liquid pressure

In a poorly conducting liquid, the currents are small and therefore magnetic fields are negligible. The equations governing the electric field are given by:

$$q = \nabla \cdot (\varepsilon\vec{E}) = \text{free charge density}$$

$$\nabla \times \vec{E} = 0$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\vec{J} = \sigma\vec{E} + q\vec{V}$$

where

ε = dielectric constant

$\vec{J}$ = current density

σ = electrical conductivity

The boundary conditions are no slip at the solid surface and the same as previously described at the liquid vapor interface.

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